Econ 131: Section 9 Social Security & Insurance

April 3, 2013

1 Problems

1. Social Security
2. Graph of Adverse Selection that sets up True/False/ Uncertain problem
3. Disability Insurance

1.1 Social Security

Consider the following model of social security. N people are born each period. Each person lives for two periods. In the first period of life, a person is young and in the second old. Thus, in any period after the initial period, half the population is young and half is old. Young people earn 2 chocolate bars while old people earn nothing. Assume that the chocolate melts, so there is no way for people to save privately.

(a) Give a brief (one sentence) rationale for government provision of social security in this model. (2.5 points)

Individuals cannot save privately since the chocolate melts so the government must provide social security so that they can eat when they are old.

Now suppose that the government of Candyland implements a social security system in the following manner. The government taxes each young person 1 chocolate bar and redistributes it to an old person in the same period. The program starts between periods 0 and 1 and ends between periods 2 and 3, as shown below. Let c_A^g denote the chocolate consumption of an agent from generation g (generation refers to the period the agent is born) at age A (age is either young or old).

(b) Under Candyland's social security program, fill in the blanks in the following chart: (10 points)

period 0		period 1	period 2		period 3
	start SS			end SS	
$c^0_{young} =$ _2_	|	$c^0_{old} =$ _1_		|	
	|	$c^1_{young} =$ _1_	$c^1_{old} =$ _1_	|	
	|		$c^2_{young} =$ _1_	|	$c^2_{old} =$ _0_

Which generation gains the most in terms of consumption? The least?

Generation 0 benefits most, being able to enjoy the benefits of the social security system without having to incur the costs of paying for a previous generation's consumption. On the other hand, generation 2 loses the most, since they pay the tax but never enjoy the benefits of the system.

(c) What assumption does this model make about the effect of social security provision on retirement behavior? Discuss how empirical evidence on retirement decisions and social security relates to this assumption. (5 POINTS)

The model assumes that the fraction of people who are "old" is invariant to the social security system. In other words, it assumes that retirement behavior is not affected by the provision of SS. In contrast, Gruber and Wise (2002) present evidence on retirement behavior around the world, and find that workers's retirement decisions are heavily influenced by whether or not additional work will increase their social security benefits.

(d) The country of Twixland is also considering implementing social security. However, the ingenious residents of Twixland have figured out a way to freeze and save chocolate for retirement. All of them have utility over consumption when young (c_A) and old (c_B) given by $\log(c_A) + \log(c_B)$. The demographic and other aspects of the economy are as in Candyland.

Replicate the chart in (b) for Twixland, assuming that the start and end of SS are completely unanticipated by its residents. (7.5 POINTS)

Since the utility is concave, individuals will want to smooth consumption perfectly across generations consume 1 chocolate bar when they are young and 1 when they are old. In Twixland, this is feasible in the absence of SS because people can save.

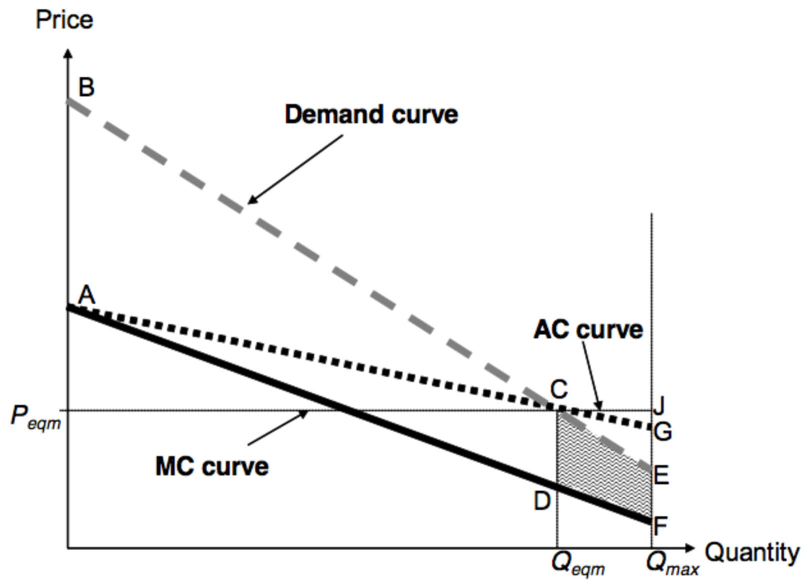
Twixlanders react to the new SS program by changing their savings decisions accordingly if they anticipate the receipt of SS when they are old. Generation 0 individuals do not anticipate getting SS when old, so they continue to save 1 chocolate bar in period 0, but gain an extra chocolate bar in period 1 because of SS. Generation 1 anticipates SS, so its consumption is unchanged (100% crowdout of government intervention). Generation 2 anticipates getting SS and therefore does not save for itself, but then gets nothing in period 3.

period 0		period 1	period 2		period 3
	start SS			end SS	
$c^0_{young} =$ _1_	|	$c^0_{old} =$ _2_		|	
	|	$c^1_{young} =$ _1_	$c^1_{old} =$ _1_	|	
	|		$c^2_{young} =$ _1_	|	$c^2_{old} =$ _0_

Again, we see that generation 0 benefits the most and generation 2 loses. The only difference relative to CandyLand is that generation 0 gains more consumption when old rather than young.

1.2 Adverse Selection

Figure 2: Adverse Selection in the Textbook Setting



Source: Einav and Finkelstein (2011).

Notes: Figure 2 shows the demand (willingness-to-pay) for a high coverage H relative to a lower coverage contract L, and the associated marginal and average incremental cost (i.e. expected insurance claims) curves. The downward sloping marginal cost curve indicates adverse selection. The efficient allocation is for everyone to be covered by H (since willingness to pay is always above marginal cost) but the equilibrium allocation covers only those whose willingness to pay is above average costs, creating the classic under-insurance result of adverse selection. The welfare loss from this under-insurance is given by the trapezoid CDEF, representing the excess of demand above marginal cost for those who are not covered by H in equilibrium.

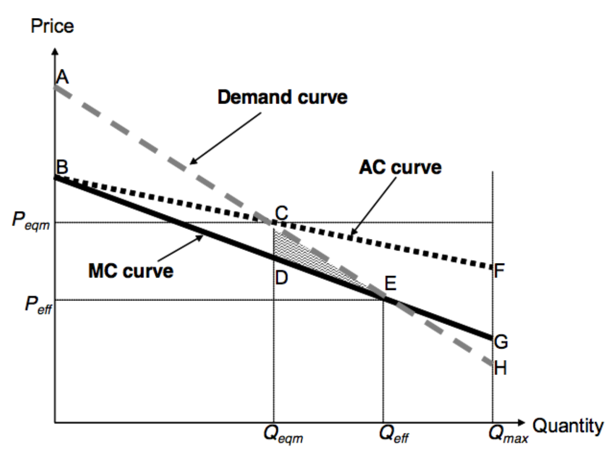
A bit more detail from Chetty and Finkelstein (2013):

"This is the key feature of adverse selection: individuals who have the highest willingness to pay for insurance are those who, on average, have the highest expected costs. This is represented in Figure 2 by drawing a downward sloping MC curve. That is, marginal cost is increasing in price and decreasing in quantity. As the price falls, the marginal individuals who select contract H have lower expected cost than infra-marginal individuals, leading to lower average costs. **The link between the demand and cost curve is arguably the most important distinction of insurance markets (or selection markets more generally) from traditional product markets.** The shape of the cost curve is driven by the demand-side consumer selection. In most other contexts, the demand curve and the cost curve are independent objects; demand is determined by preferences and costs by the production technology. The distinguishing feature of selection markets is that the demand and cost curves are tightly linked since the individuals risk type not only affects demand but also directly determines cost."

TRUE/FALSE/UNCERTAIN: Private information about risk always produces under-insurance relative to the efficient outcome and mandating insurance always improves welfare.

Uncertain. While this is true in the textbook case, large administrative costs and/or advantageous selection can make this statement incorrect. (see Figure 3 and 4 of the Handbook chapter by Chetty and Finklestein below).

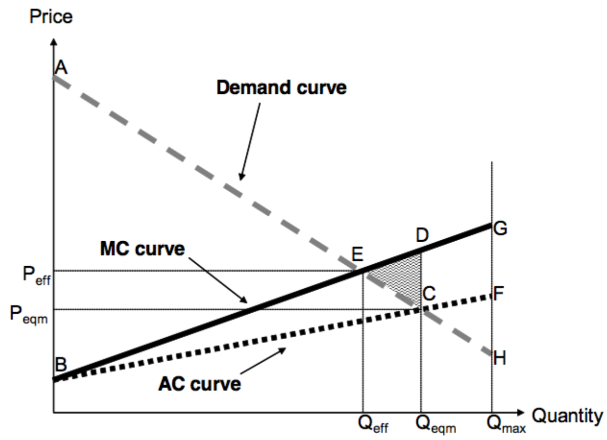
Figure 3: Adverse Selection with Additional Costs of Providing Insurance



Source: Einav, Finkelstein and Cullen (2010)

Notes: In this departure from the textbook case, we allow for the possibility of a loading factor on the insurance contract H. As a result, the marginal cost curve may now intersect the demand curve internally, in which case it is not efficient to cover all individuals with H. The efficient allocation is given by point E (where demand intersects the marginal cost curve) and the equilibrium allocation is given by point C (where demand intersects the average cost curve). Once again there is under-insurance due to adverse selection ($Q_{eqm} < Q_{eff}$) and the welfare loss from this under-insurance is given by the trianglec CDE.

Figure 4: Advantageous Selection



Source: Einav, Finkelstein and Cullen (2010).

Notes: Advantageous selection is characterized by an upward sloping marginal cost curve. The average cost curve therefore lies below the marginal cost curve, resulting in over-insurance relative to the efficient allocation ($Q_{eff} < Q_{eqm}$). The welfare loses from over-insurance is given by the shaded area CDE and represents the excess of marginal cost over willingness to pay for people whose willingness to pay exceed the average costs of those covered by H.

1.3 Disability Insurance

Consider an economy where there are three types of people who want to buy disability insurance. Each type has the same health-based risks. They each have a 2 percent chance of being incapacitated due to health risks which are uncorrelated with their risk taking behavior. But the people differ in their hobbies and work. The risk averse types walk to work and have very low risk hobbies. Their outside risk of being incapacitated is 3 percent. The risk neutral type drives to work and actively plays soccer on the weekends. Therefore, their non health risk of being incapacitated is 8 percent. The third type loves risk. They are the fire fighters and sky dive on weekends. Therefore, their outside risk of being incapacitated is 88 percent. There are equal numbers of each type. Long-term care insurance provides income if they are incapacitated for the rest of their life (there are no additional costs). Individuals have the following utility function over consumption (or income):

$$u(c) = \log(c)$$

Individuals earn \$500 if healthy, and \$10 if incapacitated and have a time discount rate $\beta = .8$.

a) If the insurance company can differentiate the types, what is the socially optimal level of insurance for each type. (Note, no math is required for this question.)

b) What is the actuarially fair price for the insurance for each group? What is the expected utility of each group given that price if they buy insurance? What is the expected utility if they do not buy insurance?

c) i) Now assume that the insurance company is not able to differentiate between the three types. Therefore, it offers a policy that fully insures individuals at the same price to all three types. Assuming all three types buy this insurance policy, what is the price for insurance?

ii) Prove that with the price you found in part c, the risk averse would not be willing to buy the full insurance policy (build on part b).

d) Since the risk averse do not buy the policy, the insurance company cannot offer the policy at the price from part c. What is the new price for this policy if the risk averse drop out of the market? Who will buy at this price (show mathematically)?

e) You have demonstrated an example of a market unraveling. Explain the intuition for why it happened.

ANSWER

a) If the insurance company can differentiate the types, what is the social optimum level of insurance. (Note, no math is required for this question.)
All agents have concave utility functions (the first derivative of the utility function is positive and the second derivative is negative i.e. they have diminishing marginal utility) and are therefore risk averse. Risk averse agents fully insure when offered actuarially fair insurance, therefore all three types will choose to purchase insurance policies.

b) i) What is the actuarially fair price for the insurance for each group? What is the expected utility of each group?
Actuarily Fair Price:

$$p_g = \frac{(y_2) * \Pr(disability)}{\beta}$$

Therefore, for the risk adverse:

$$\begin{aligned} p_g &= \frac{(500) * .05}{.8} \\ &= 31.25 \end{aligned}$$

Therefore, for the risk neutral:

$$\begin{aligned} p_g &= \frac{(500) * .1}{.8} \\ &= 62.5 \end{aligned}$$

And for the risk loving:

$$\begin{aligned} p_g &= \frac{(500) * .6}{.8} \\ &= 375 \end{aligned}$$

Expected utility of each group if they buy insurance:

$$\begin{aligned} \mathrm{E}[U] &= \mathrm{E}[\log(y_1) + \beta \log(y_2)] \\ &= \log(w - p_g) + \beta * log(w) \\ &= \log(w - \frac{(y_2) * \Pr(disability)}{\beta}) + \beta * log(w) \end{aligned}$$

For the risk adverse:

$$\begin{aligned} \mathrm{E}(U) &= \log(w - p_g) + \beta * log(w) \\ &= \log(500 - 31.25) + .8 * log(500) \\ &= 11.12 \end{aligned}$$

or the risk neutral:

$$\begin{aligned} \mathrm{E}(U) &= \log(w - p_g) + \beta * log(w) \\ &= \log(500 - 62.5) + .8 * log(500) \\ &= 11.05 \end{aligned}$$

or the risk loving:

$$\begin{aligned} \mathrm{E}(U) &= \log(w - p_g) + \beta * log(w) \\ &= \log(500 - 375) + .8 * log(500) \\ &= 9.80 \end{aligned}$$

Expected Utility of each group if they do not buy insurance:

$$\begin{aligned} \mathrm{E}[U] &= \mathrm{E}[\log(y_1) + \beta \log(y_2)] \\ &= \log(w) + \beta(\Pr(disability) * \log(w_2) + (1 - \Pr(disability)) * \log(w)) \\ &= (1 + \beta - \beta * \Pr(disability)) \log(w) + \beta * \Pr(disability) * \log(w_2) \end{aligned}$$

For the risk adverse:

$$\begin{aligned} \mathrm{E}(U) &= (1 + \beta - \beta * \Pr(disability)) \log(w) + \beta * \Pr(disability) * \log(w_2) \\ &= (1 + .8 - .8 * .05) * log(500) + .8 * .05 * \log(10) \\ &= 11.03 \end{aligned}$$

For the risk neutral:

$$\begin{aligned} \mathrm{E}(U) &= (1 + \beta - \beta * \Pr(disability)) \log(w) + \beta * \Pr(disability) * \log(w_2) \\ &= (1 + .8 - .8 * .1) * log(500) + .8 * .1 * \log(10) \\ &= 10.87 \end{aligned}$$

For the risk loving:

$$\begin{aligned}
\mathrm{E}(U) &= (1+\beta-\beta*\Pr(disability))\log(w)+\beta*\Pr(disability)*\log(w_2)\\
&= (1+.8-.8*.6)*log(500)+.8*.6*\log(10)\\
&= 9.31
\end{aligned}$$

ii) What prices are each groups willing to pay for the insurance? To be willing to pay for insurance, the expected utility from life with the insurance must be better than the expected utility from life without the insurance.

$$\begin{aligned}
\log(w-p)+\beta\log(w) &\geq (1+\beta-\beta*\Pr(disability))\log(w)+\beta*\Pr(disability)*\log(w_2)\\
\log(500-p) &\geq (1-.8*\Pr(disability))\log(500)+.8**\Pr(disability)*\log(10)\\
500-p &\geq 500^{1-.8*\Pr(disability)}10^{.8*\Pr(disability)}\\
p &\leq 500-500^{1-.8*\Pr(disability)}10^{.8*\Pr(disability)}
\end{aligned}$$

Therefore, the maximum the risk adverse type is willing to pay is:

$$\begin{aligned}
p &\leq 500-500^{1-.8*\Pr(disability)}10^{.8*\Pr(disability)}\\
&= 500-500^{.96}*10^{.04}\\
&= 108.95
\end{aligned}$$

Therefore, the maximum the risk neutral type is willing to pay is:

$$\begin{aligned}
p &\leq 500-500^{1-.8*\Pr(disability)}10^{.8*\Pr(disability)}\\
&= 500-500^{.92}*10^{.08}\\
&= 194.67
\end{aligned}$$

Therefore, the maximum the risk loving type is willing to pay is:

$$\begin{aligned}
p &\leq 500-500^{1-.8*\Pr(disability)}10^{.8*\Pr(disability)}\\
&= 500-500^{.6}*10^{.4}\\
&= 471.66
\end{aligned}$$

c) i) Now assume that the insurance company is not able to differentiate between the three types. Therefore, it has to offer the policy at the same price to all three types. Assuming all three types by the insurance, what is the price for insurance?
The actuarily fair price is:

$$\begin{aligned}
p_a &= \frac{(y_2)*\Pr(disability)}{\beta}\\
&= \frac{500*(.05+.1+.6)/3}{.8}\\
&= 156.25
\end{aligned}$$

ii) Prove that with the price you found in part c, the risk adverse would not be willing to buy insurance. (See part b)ii)).

As you can see from part c, the price of 135.42 is greater than the maximum price that the risk adverse type is willing to pay (108.95). Therefore, the risk adverse people will not buy the policy.

d) Since the risk adverse do not buy the policy, the insurance company cannot offer the policy at the price from part c. What is the new price? Who will buy at the price and prove it?

The actuarily fair price is:

$$
\begin{aligned}
p_a &= \frac{(y_2) * \Pr(disability)}{\beta} \\
&= \frac{500 * (.1 + .6)/2}{.8} \\
&= 218.75
\end{aligned}
$$

Therefore, only the risk loving will buy and the market has unraveled.

e) You have demonstrated an example of a market unraveling. Explain the intuition for why it happened.

This market unraveling is called adverse selection. It is the result of asymmetric information between the insurance companies and those it may insure. The insurance company cannot distinguish between risk preferences, so it must offer the same policy to everyone, but in order to earn zero profit they must charge a premium equal to the population risk times the coverage amount. Since the risk adverse know who they are, they will not insure in order to maximize utility. This causes the risk of accident among the insured population to increase, causing fewer people to insure, and so on.

f) If the individual had no income if they were disabled, how would your results change?

If the individuals had no income if they were disabled, the individuals would have utility of negative infinity if they were disabled (due to the log nature of the problem). Therefore, the individuals would be willing to pay almost anything to buy insurance. Therefore, even the risk adverse type would buy insurance at whatever price. The market would not unravel.