

## **1 Conceptual Topics**

1. Tiebout Model

## **2 Problems**

1. Voting and Police Protection
2. Swimming Pool
3. National Park
4. Big Dig

## 2.1 Voting and Police Protection

**Carrboro has three equal-sized groups of people: (1) type A people consistently prefer more police protection to less; (2) type B people prefer high levels of police protection to low levels and they prefer low levels to medium levels; (3) type C people prefer medium levels to low levels, which they in turn prefer by a modest amount to high levels.**

**a. Which types of people have single-peaked preferences? Which have multi-peaked preferences?**

Types A and C have single-peaked preferences, with peaks at “high” and “medium” respectively. Type B has multiple-peaked preferences, with peaks at “high” and “low” and a dip at “medium.”

**b. Will majority voting generate consistent outcomes in this case? Why or why not?**

Majority voting does not usually generate consistent outcomes when some voters have preferences that fail to be single peaked. But they do happen to generate consistent outcomes in this case. If “high” and “low” are the two options on the ballot, “high” will win, since types A and B will vote for it. Similarly “high” wins when “high” and “medium” are the two options on the ballot. When “low” and “medium” are on the ballot, “medium” wins, since types A and C will vote for it. Finally, when all three are on the ballot, types A and B will both vote for “high,” which will therefore win. Notice that there are no cycles, so the voting outcomes are, in fact, consistent. The decisions coincide with those that would be made by a society that prefers “high” to “medium” and “medium” to “low.”

## 2.2 Swimming Pool

The city of Gruberville is considering whether to build a new public swimming pool. This pool would have a capacity of 800 swimmers per day, and the proposed admission fee is \$6 per swimmer per day. The estimated cost of the swimming pool, averaged over the life of the pool, is \$4 per swimmer per day.

Gruberville has hired you to assess this project. Fortunately, the neighboring identical town of Figliona already has a pool, and the town has randomly varied the price of that pool to find how price affects usage. The results from their study follow:

Swimming pool price per day	Number of swimmers per day
\$8	500
\$10	200
\$4	1,100
\$6	800
\$2	1,400

- a. **If the swimming pool is built as planned, what would be the net benefit per day from the swimming pool? What is the consumer surplus for swimmers?**

At an admission fee of \$6, the city earns a profit of \$2 per swimmer per day, or a total of \$1,600 per day. Consumer surplus can be determined from the demand function. With every \$2 increase in price, quantity demanded falls by 300. If you assume a linear demand function, quantity demanded will be zero at an admission price of \$11.33. The triangle of consumer surplus is bounded by the quantity of 800 and the vertical distance of  $\$11.33 - \$6 = \$5.33$ .

Consumer surplus =  $\frac{1}{2} (800 \times 5.33) = \$2,132$ . Total surplus ( $\$1,600 + \$2,132$ ) is \$3,732 per day.

- b. **Given this information, is an 800-swimmer pool the optimally sized pool for Gruberville to build? Explain.**

If you assume that the cost per swimmer does not vary with the size of the pool, then this is not the optimal pool size. Optimality occurs where marginal cost equals marginal benefit. Since marginal cost is \$4, the pool should set a price of \$4 to swim. Then the marginal benefit to additional swimmers will be exactly \$4 (the last swimmer was just willing to pay to get in). There will be 1,100 swimmers at this price, so the optimum pool size is thus 1,100. The town earns no profits on the pool, but the consumer surplus now becomes  $\frac{1}{2} (1,100 \times 7.33) = \$4,031.50$  per day.

### 2.3 National Park

Jellystone National Park is located 10 minutes away from city A and 20 minutes away from city B. Cities A and B have 200,000 inhabitants each, and residents in both cities have the same income and preferences for national parks. Assume that the cost for an individual to go to a national park is represented by the cost of the time it takes her to get into the park. Also assume that the cost of time for individuals in cities A and B is \$.50 per minute.

You observe that each inhabitant of city A goes to Jellystone ten times a year while each inhabitant of city B goes only five times a year. Assume the following: the only people who go to the park are the residents of cities A and B; the cost of running Jellystone is \$1,500,000 a year; and the social discount rate is 10%. Also assume that the park lasts forever.

**a. Compute the cost per visit to Jellystone for an inhabitant of each city.**

A day at the park will cost a resident of city A \$10 (10 minutes each way  $\times$  \$.50 per minute) and a resident of city B \$20 (20 minutes each way  $\times$  \$.50 per minute).

**b. Assuming that those two observations (cost per visit and number of visits per inhabitant of city A, and cost per visit and number of visits per inhabitant of city B) correspond to two points of the same linear individual demand curve for visits to Jellystone, derive that demand curve. What is the consumer surplus for inhabitants of each city? What is the total consumer surplus?**

City A residents account for 2,000,000 visits at a price of \$10; city B residents account for 1,000,000 visits at a price of \$20. If the demand function is linear, every \$10 price increase is associated with a decrease in quantity of 1,000,000 visits. Thus, the demand function is  $\text{Price} = \$30 - 0.00001Q$ .

City A residents have a consumer surplus of  $\frac{1}{2} (2,000,000 \times \$20) = \$20,000,000$ .

City B residents have a consumer surplus of  $\frac{1}{2} (1,000,000 \times \$10) = \$5,000,000$ .

Total consumer surplus is \$25,000,000.

**c. There is a timber developer who wants to buy Jellystone to run his business. He is offering \$100 million for the park. Should the park be sold?**

Each year consumer surplus is \$25 million and operating costs are \$1.5 million, for a net benefit of \$23.5 million. Applying the social discount rate of 10% yields a PDV of \$235 million, much more than the timber developer's offer. The park should not be sold.

## 2.4 Big Dig

15. Imagine you are the governor of Massachusetts 15 years ago and need to decide if you should support the “Big Dig” highway and bridge construction project.

The Big Dig is estimated to take 7 years to complete. The project will require \$45 million in construction materials per year and \$20 million in labor costs per year. In addition, the construction will disrupt transportation within the city for the duration of the construction. The transportation disruption lengthens transport times for 100,000 workers by 30 hours a year. All workers are paid \$15 per hour (assume that there are no distortions and that the wage reflects each worker’s per-hour valuation of leisure).

The Big Dig, when finished, will ease transportation within the city. Each of the 100,000 workers will have their transport time reduced by 35 hours a year as compared to the preconstruction transport time. In addition, part of the Big Dig project involves converting the space formerly taken up by an elevated highway into a large park. The State of Massachusetts has determined that each worker will value the park at \$40 per year. We will assume that no one else will use the park.

We also assume the government has a 5% discount rate and that the workers live forever. The benefits to the Big Dig begin in year 7, assuming the project begins in year 0 (i.e., the project runs for 7 years, from  $t = 0$  to  $t = 6$ ).

- a. Should you, as the governor, proceed with the project? Formally show the cost-benefit analysis.

Costs during construction are as follows:

Year	Materials	Labor	Delay costs	Total	Total discounted to present
0	\$45m	\$20m	$100,000 \times 30 \times \$15 = \$45m$	\$110m	\$110.00m
1	45m	\$20m	45m	110m	$110m / (1.05) = 104.76m$
2	45m	\$20m	45m	110m	$110m / (1.05^2) = 99.77m$
3	45m	\$20m	45m	110m	95.02m
4	45m	\$20m	45m	110m	90.50m
5	45m	\$20m	45m	110m	86.19m
6	45m	\$20m	45m	110m	82.08m
<b>Total Costs</b>					<b>\$668.33m</b>



The value of the benefits for the park are  $\$40(100,000) = \$4$  million per year, and the value of saved time is  $100,000 \times 35 \times \$15 = \$52.5$  million. The per-year benefit from the project is thus  $\$56.5$  million, starting in year 7. The present value of the benefit, as of year 0, with a 5% discount rate is thus  $(\$56.5m/0.05)/(1.05^7) = \$803$  million. Since this is bigger than the discounted costs, the project is worthwhile.

- b. It occurs to you, after completing the calculation in part a, that it is possible the cost estimates are uncertain. If the construction materials estimate is \$45 million with 50% probability and \$100 million with 50% probability, should the project proceed? Assume that the government is risk neutral.**

The new information suggests that the expected value of the materials cost is instead  $.5(\$45m) + .5(\$100m) = \$72.5$  million per year for each of the first 7 years. The new total expected cost per year is  $\$72.5m + \$20m + \$45m = \$137.5m$ . Using Excel, where the cell formula for PDV is  $=A2/(1.05^B2)$ :

Principal	Years	PDV
137.5	0	\$137.5
137.5	1	\$130.9524
137.5	2	\$124.7166
137.5	3	\$118.7777
137.5	4	\$113.1216
137.5	5	\$107.7348
137.5	6	\$102.6046
	<b>Total</b>	<b>\$835.4077</b>

Given the 50% probability that material costs will be \$100 million instead of the original estimate of \$45 million, the expected costs exceed the expected benefits and the project should not be undertaken.

Students who set up this project using a spreadsheet can see that it is easy to test varying assumptions without tedious recalculation.